

Probability models & Expected Value**Example 1:** I have a spinner. It has 5 colors:

Red has 100 degrees and you win \$2

Green has 60 degrees and you lose \$2

Yellow has 65 degrees and you win \$1

Blue has 90 degrees and you lose \$1

Orange has 45 degrees and you win \$4

- What colors do you expect to spin the most? Least?
- Overall for this spinner, do you expect to win money or lose money? Why?
- Create a probability model for this spinner

X	\$2	-\$2	\$1	-\$1	\$4
P(X)	$\frac{100}{360}$	$\frac{60}{360}$	$\frac{65}{360}$	$\frac{90}{360}$	$\frac{45}{360}$
	27.8%	16.7%	18.1%	25%	12.5%

EXPECTED VALUE:

What is it?

- Basically ... Long run average
- Also called ... mean, average

• amount gain on one play/trial (on average)

- Symbol: $E(X)$ = Expected Value of X

$$E(X) = \$0.56$$

Formula (how to find it):

- X is a variable with the following probability model:

X	X1	X2	X3 ...	Xn
P(X)	P(X1)	P(X2)	P(X3) ...	P(Xn)

- Find the mean (expected value) by doing the following:

$$E(X) = \sum (X_i * P(X_i))$$

sum

Class Grade

HW 15% 75

CW 25% 80

T 60% 78

Example:

X	0	1	2	3	4	5
P(X)	0.05	0.12	0.18	0.2	0.4	0.05

What is the mean (expected value)?

$$2.186$$

$$2.93$$

Let's go back to the spinner from before...

Red has 100 degrees and you win \$2

Green has 60 degrees and you lose \$2

Yellow has 65 degrees and you win \$1

Blue has 90 degrees and you lose \$1

Orange has 45 degrees and you win \$4

Probability Model:

	RED(100°)	BLUE(90°)	GREEN(60°)	ORANGE(45°)	YELLOW(65°)
Value	2	-1	-2	4	1
P(Value)	0.2778	0.25	0.1667	0.125	0.1806

- Find the expected value of the spinner

$$E(X) = \$0.65$$

$$E(V)$$

- So if I wanted to find out how much I EXPECT to win on 65 spins, I would...

$$65 \times 0.65 = \$42.25$$

Using the data from the spinner, use your calculator to find the Expected Value (mean):

- Data goes in... $L_1 = X$

$$L_2 = P(X)$$

- Use:

1 var stats L_1, L_2

- Mean:

$$\bar{X} = E(X)$$

Using the following data (and the calculator) to find the Expected Value (mean):

Example:

X	-10	5	-15	10	20	-5
P(X)	0.15	0.10	0.17	0.21	0.3	0.07

$$E(X) = 4.2$$

Example 2: I play a game where I roll a die, and depending on what face comes up, I win/lose money.

- Win \$5 for rolling a 2 or a 3
- Lose \$4 for rolling a 6
- Win \$1 for rolling a 1
- Lose \$1 for rolling a 4 or 5

- Create a probability model for this dice

X	\$5	-\$4	1	-\$1
P(X)	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$

- Find the expected value of this game

$$E(X) = \$0.83$$

- If I played the game 35 times, how much do I expect to win?

$$0.83 \times 35 = \$29.05$$

Example 3:

Let's play a game. You pay \$5 to play. In the game, you get to draw one card from the deck.

- If you draw the ace of hearts, you win \$100
- If you draw any other ace, you win \$15
- If you draw any other heart, you win \$10
- If you draw any black card, you win \$5
- Any other card, you win nothing

$P(95 \text{ or } 10 \text{ or } 5)$
 $P(\text{gain } \$)$

a) Create the probability model below for the GAIN

X	95	10	5	0	-5
P(X)	$\frac{1}{52}$	$\frac{3}{52}$	$\frac{12}{52}$	$\frac{24}{52}$	$\frac{12}{52}$

b) What is the chance that you get some money?

$$P(X \geq 5) = \frac{16}{52}$$

c) What is the chance that you gain at least \$5?

$$P(X \geq 5) =$$

d) What is the probability of gaining at least \$10?

$$P(X \geq 10) = \frac{4}{52}$$

e) What is the probability of gaining more than \$20?

$$P(X > 20) = \frac{1}{52}$$

f) If I play the game 25 times, how much do I expect to gain?

$$E(X) = \$2.40 \times 25 = \$60$$

Example 4: You are at the county fair and decide to try a game of ring toss. You know that you have a probability at any time of throwing the ring on the bottle of 15%. You get three rings to toss for \$5. When you make the ring, you stop playing. If you make the first ring you win \$20, if you make the second one you get \$10, and if you make the last one you win \$5.

a. Use a tree diagram to help you outline the sample space for this scenario

b. Create the probability model for your winnings.

c. What is the probability that you will win at least some money?

d. What is the expected winnings?

Complete the worksheet #1 ~~6~~

①

#5

X	60,000	45,000	15,000
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

②

$$E(X) = 47,500 \text{ people}$$

③

$$8 \times 47,500 = 380,000$$

Worksheet answers:

$$1) E(X) = 7.86$$

2)

X	\$74,999	-\$1
P(X)	$\frac{1}{10,000}$	$\frac{9,999}{10,000}$

⑥ $E(X) = \$6.50$

X	\$74,999	\$999	\$99	-\$1
P(X)	$\frac{1}{10,000}$	$\frac{5}{10,000}$	$\frac{10}{10,000}$	$\frac{9984}{10,000}$

$$⑦ E(X) = \$7.10$$

④ ①

X	250,000	-10,000
P(X)	0.1	0.9

X	800,000	-20,000
P(X)	0.05	0.95

X	40,000	-2,000
P(X)	0.5	0.5

- (b) A = \$16,000
 B = \$19,000
 C = \$21,000

X	60,000	45,000	15,000
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$E(X) = 47,500 \text{ people}$$

#6)

(a) Game 1

(b) $E(X1) =$

X	-1	0	1
P(x)	0.40	0.10	0.50

\$ 0.10

Game 2

(c) $E(X2) =$

X	-1	0	1
P(x)	0.05	0.80	0.15

\$ 0.10

② \$20

② \$20